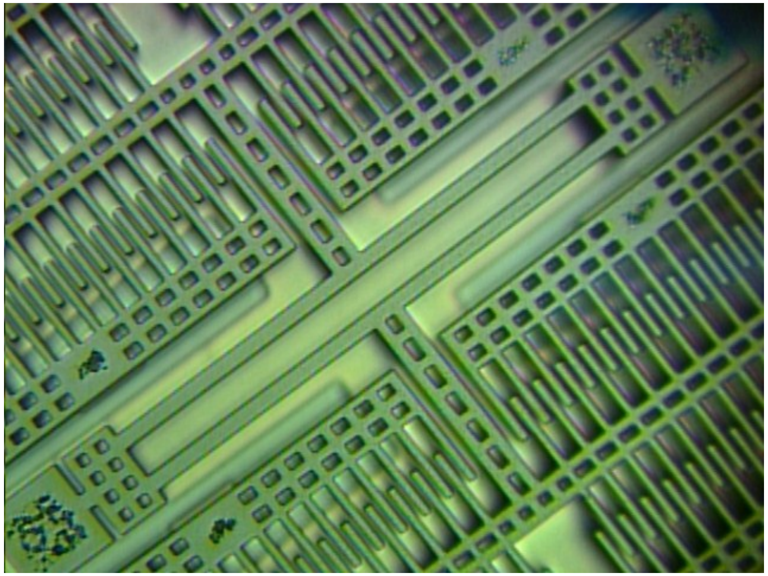
Resonant Strain Sensor



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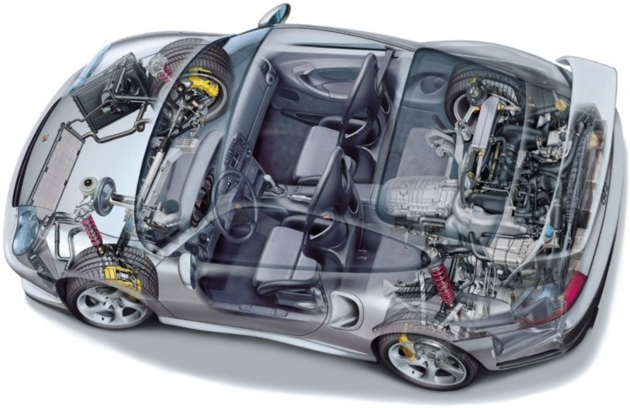
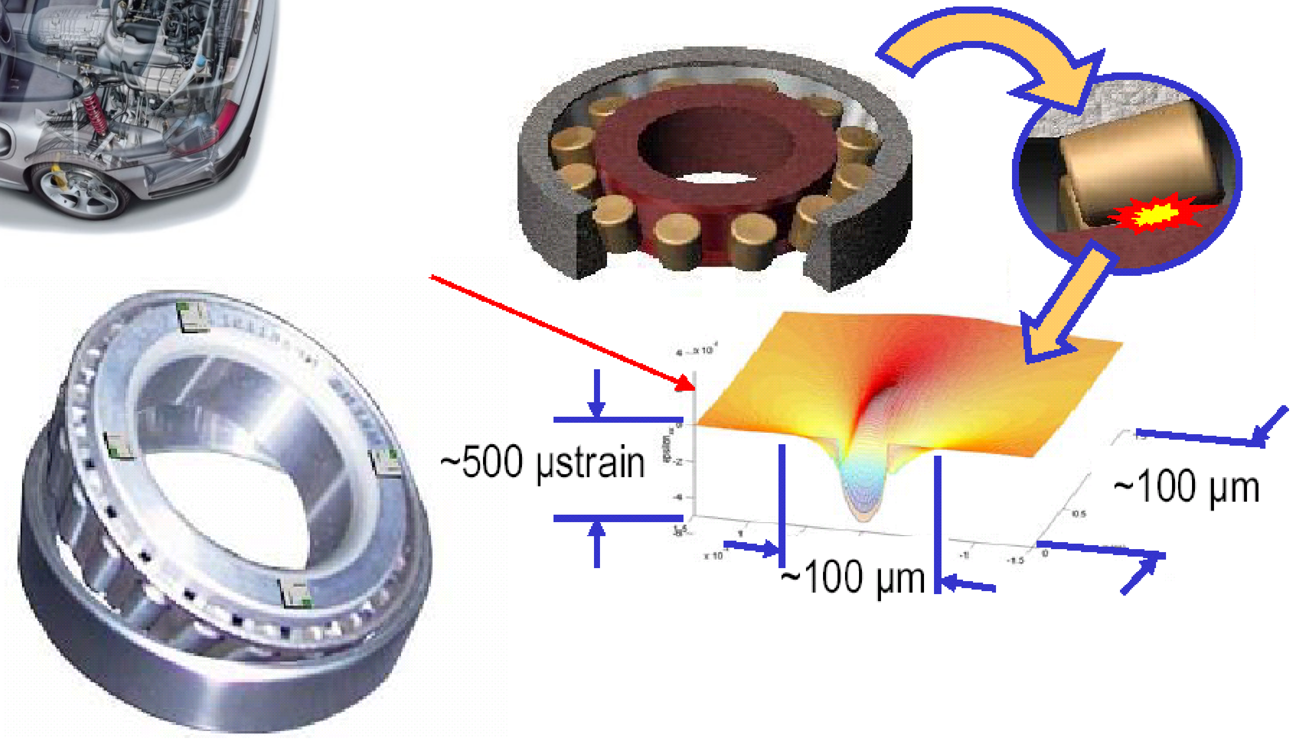
Outline

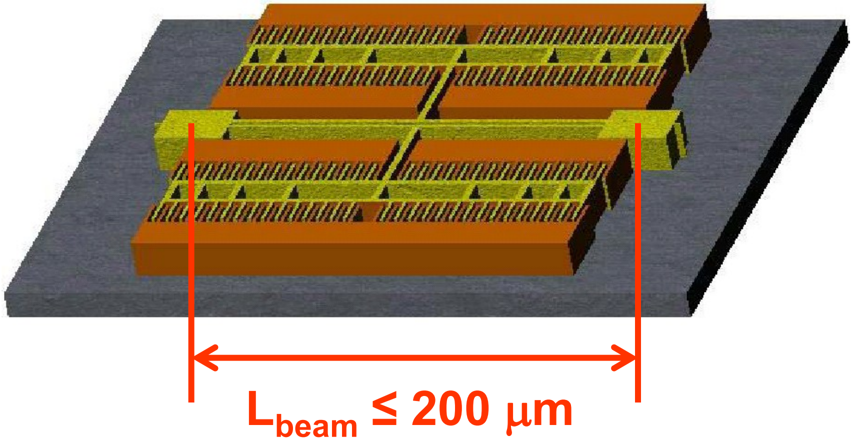
- Strain sensor
- Resonant force sensing
- Oscillator analysis
  - Phase and frequency noise
  - Design for minimum noise
- Oscillator sustaining circuit
- Frequency-to-digital conversion
- Conclusions

Strain Sensor Applications





Strain Sensor Specifications

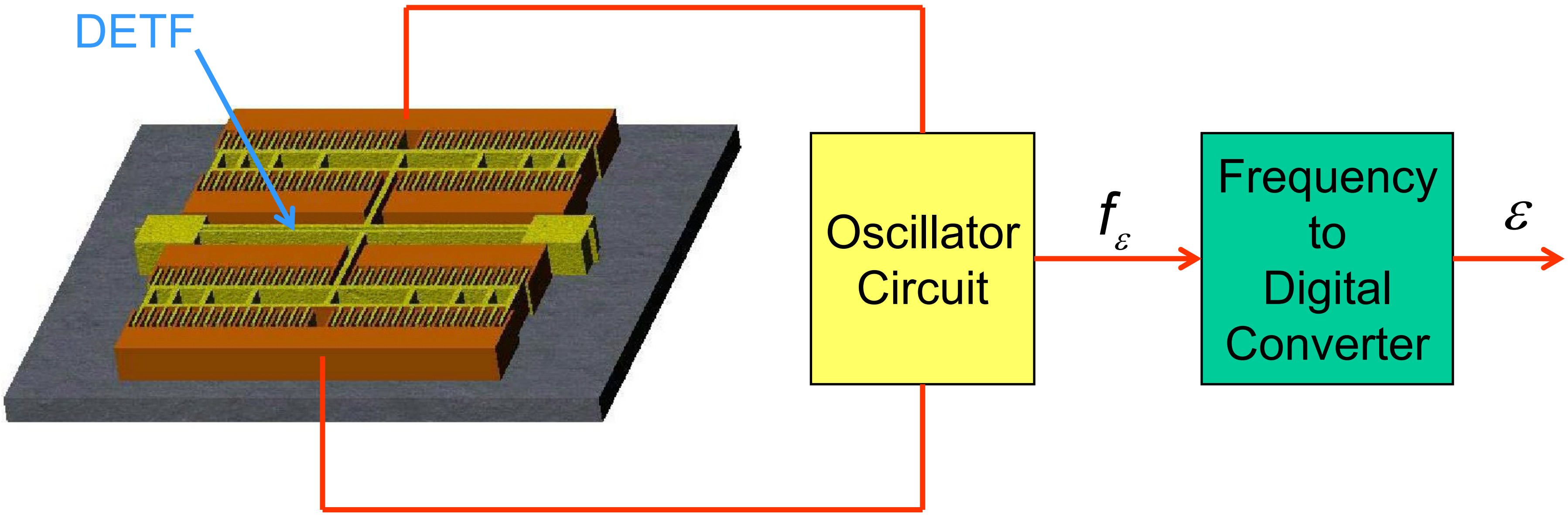


- Resolution: 0.1 με
- Full-scale: ±1000 με
- Bandwidth: 10 kHz
- Gauge length: ≤200 μm

Strain: $\varepsilon = \dfrac{\Delta L_g}{L_g}$

$$\begin{aligned}\Delta L_g &= \varepsilon \times L_g \\ &= 0.1\mu \times 200\mu\text{m} \\ &= 20\text{pm} \\ &= \underline{\underline{0.2 \times 10^{-10}\text{m}}}\end{aligned}$$

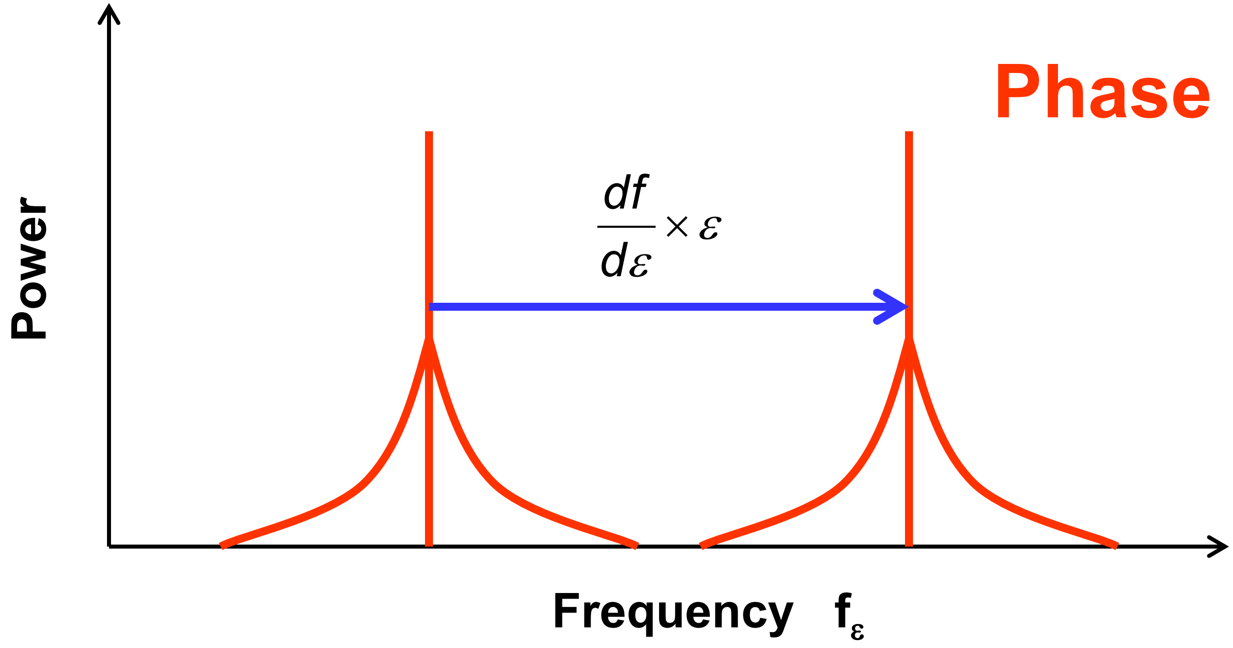
Resonant Strain Sensor



Frequency Modulation: $f_\varepsilon = f_o + \dfrac{df}{d\varepsilon} \times \varepsilon$

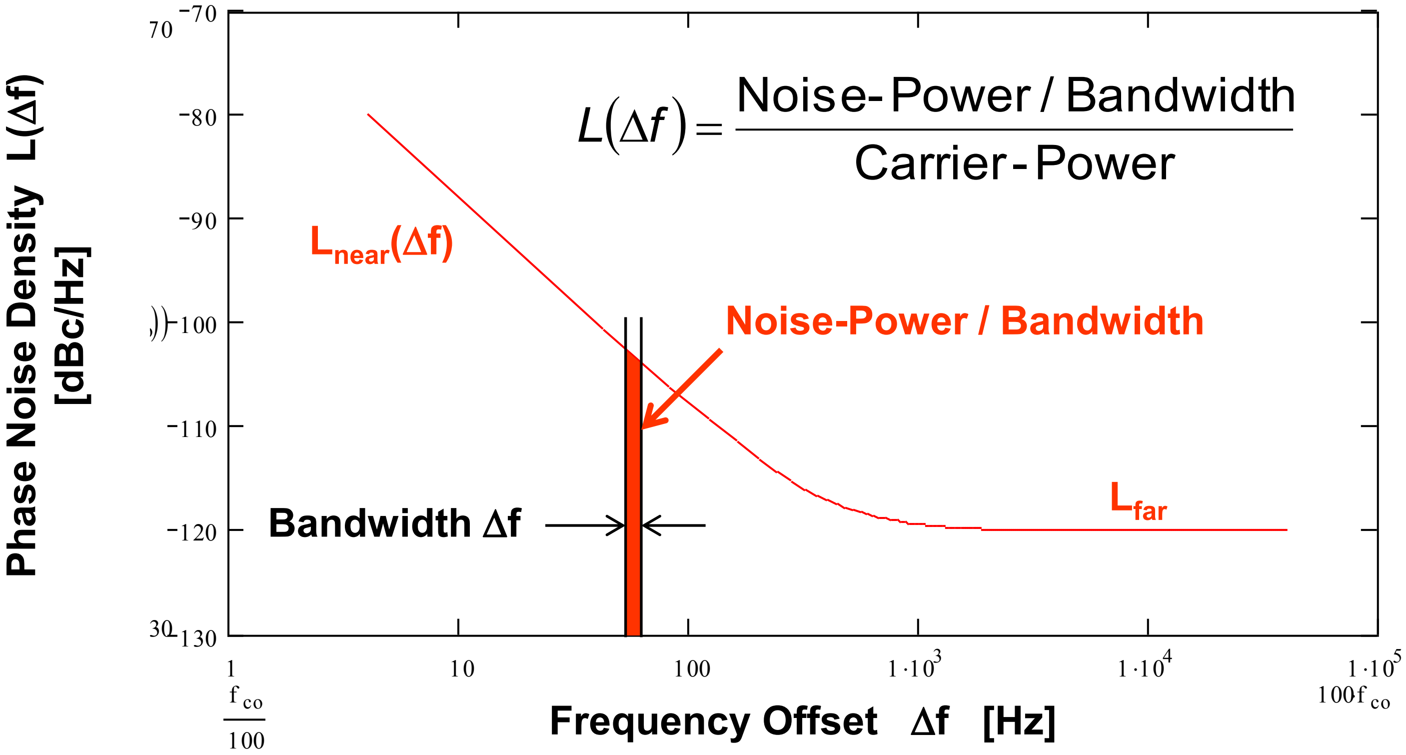
This design: $f_o = 217\text{kHz}$ $\dfrac{df}{d\varepsilon} = 38\text{Hz}/\mu\varepsilon$

Frequency Modulation



Phase Noise!

Phase Noise



$$L(\Delta f) = \frac{\text{Noise-Power / Bandwidth}}{\text{Carrier-Power}}$$

Frequency Noise

Frequency Noise $S_f(\Delta f) = 2 \times (\Delta f)^2 \times L(\Delta f)$

Total Noise

$$S_{fT} = 2\int^{B} S_f(\Delta f)\, d\Delta f$$

$$\cong 2\int^{B} (\Delta f)^2 L_{far}\, d\Delta f \qquad \text{for} \quad B >> f_{co}$$

$$= \frac{2}{3} B^3 L_{far}$$

Phase Noise

$$L_{far} \le 1.5 \times \frac{S_{fT}}{B^3}$$

$$= 1.5 \times \frac{(3.8\text{Hz})^2}{2 \times (10\text{kHz})^2} = \underline{\underline{-110\text{dBc/Hz}}}$$

Prior Art MEMS Resonators

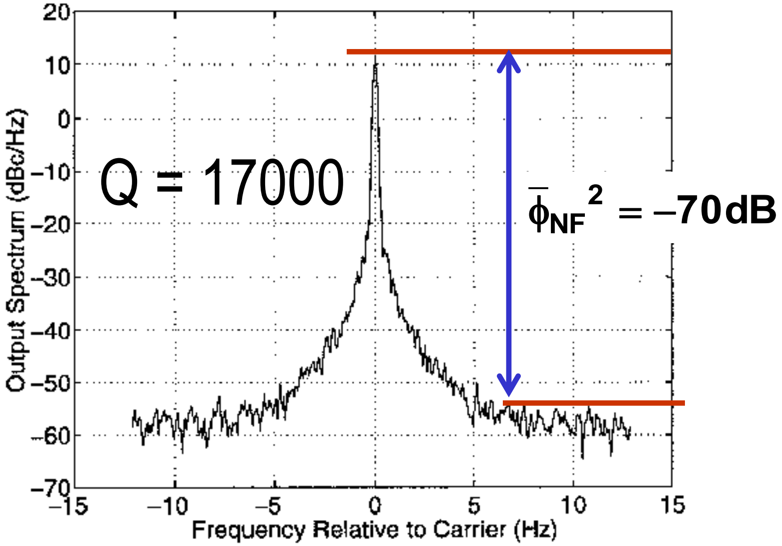
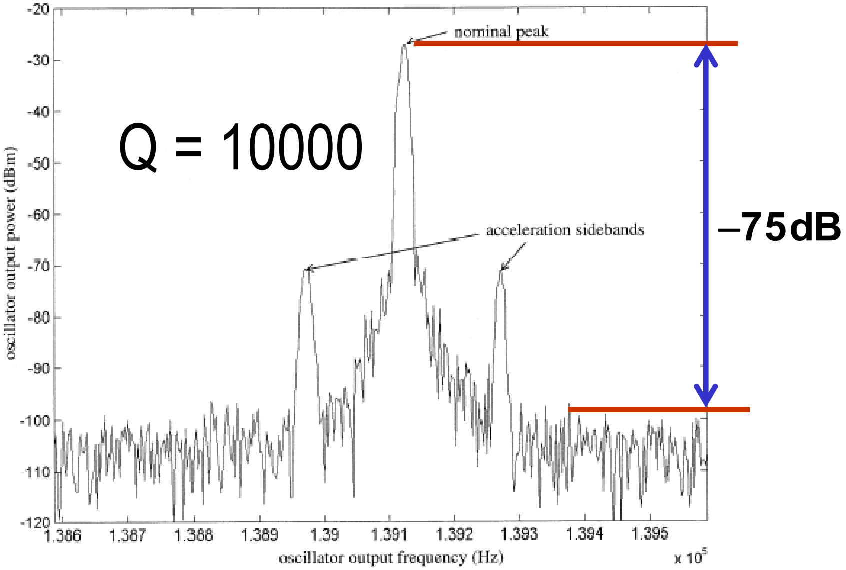


Figure 5. Output spectrum of oscillator

[T. Roessig , 1997]



[A. Seshia , 2002]

- Vacuum encapsulation
- Integrated electronics
- Much worse than quartz – why?

MEMS Oscillator Noise

	Roessig 1997	Nguyen 1999	Seshia 2002	Seth 2004	Goal (This Work)
Noise L_{far} [dBc/Hz]	**-70**	**-70**	**-75**	**-83**	**< -110**
Vacuum	✓	✓	✓	✓	✗
Monolithic	✓	✓	✓	✗	✗
Frequency, f_o	175 kHz	16.5 kHz	145 kHz	57 kHz	217 kHz
Motional Res., R_x	?	3.8 MΩ	?	2.5 MΩ	
Q	17000	51000	10000	14000	
Amplitude V_{Drive}	?	~ 34 mV	~36 mV	5.3 mV	

Need oscillator with 1000x lower noise than prior art!

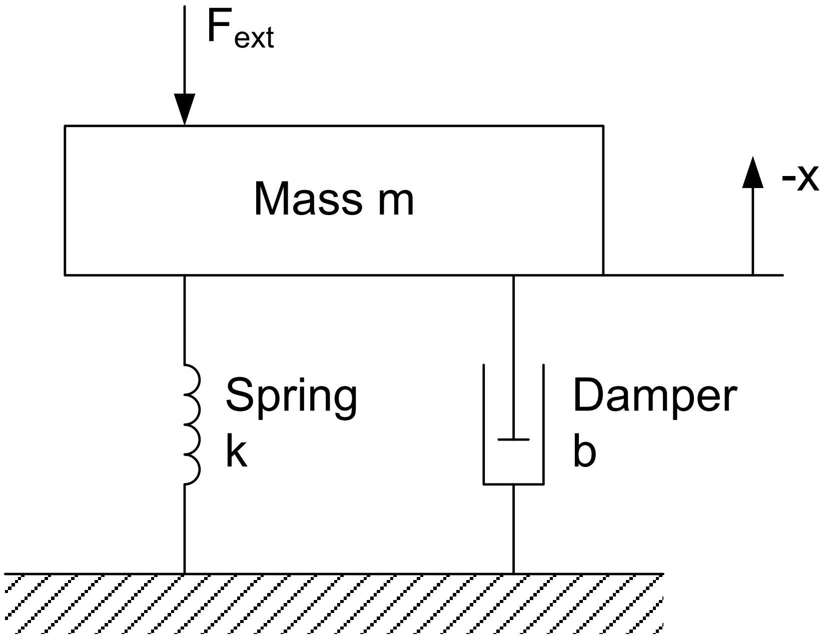
Low-Noise Oscillator Design

$$L(\Delta f) = \frac{\text{Noise-Power / Bandwidth}}{\text{Carrier-Power}} \begin{matrix} \longleftarrow \text{small} \\ \longleftarrow \textbf{BIG!} \end{matrix}$$

Constrained by gap, nonlinearity

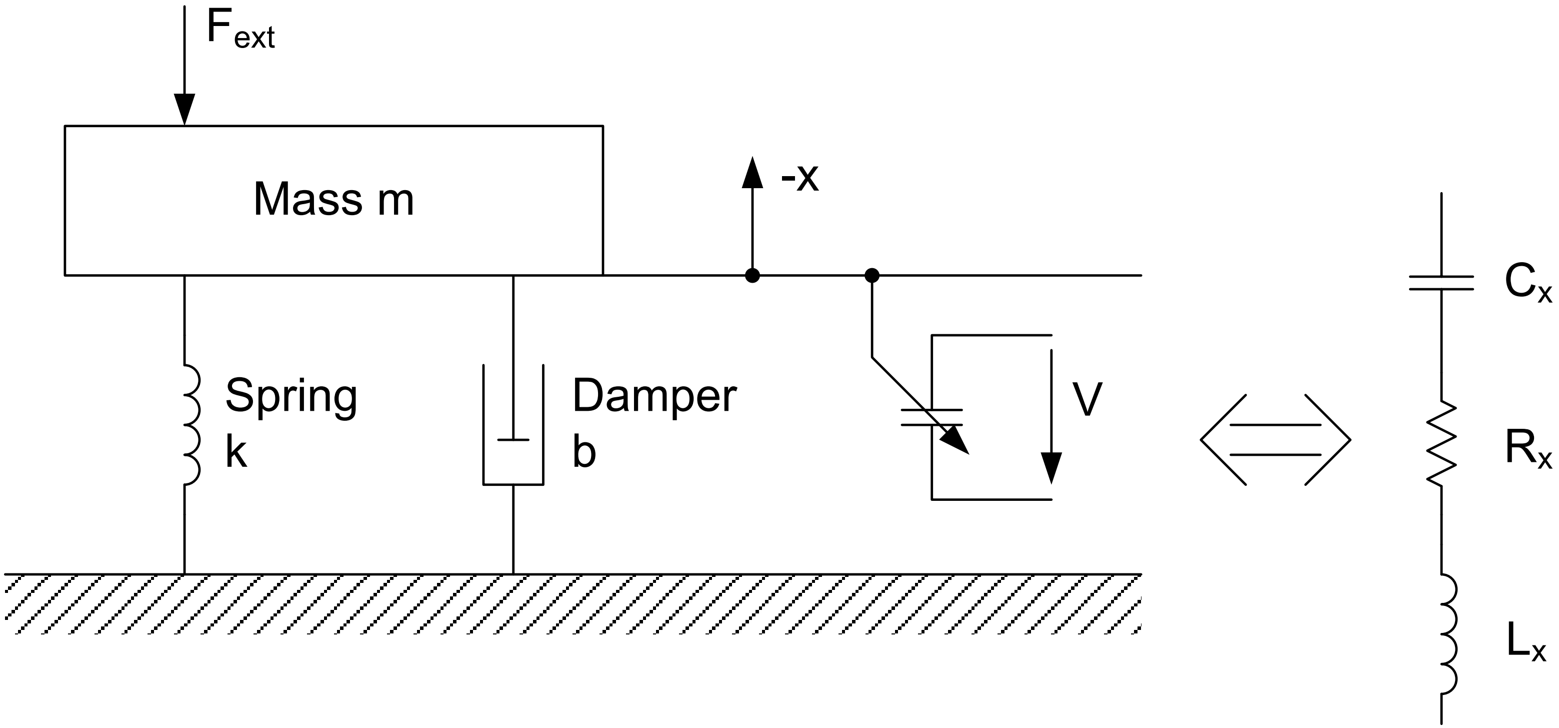
What is the noise power?

Electro-Mechanical Oscillator



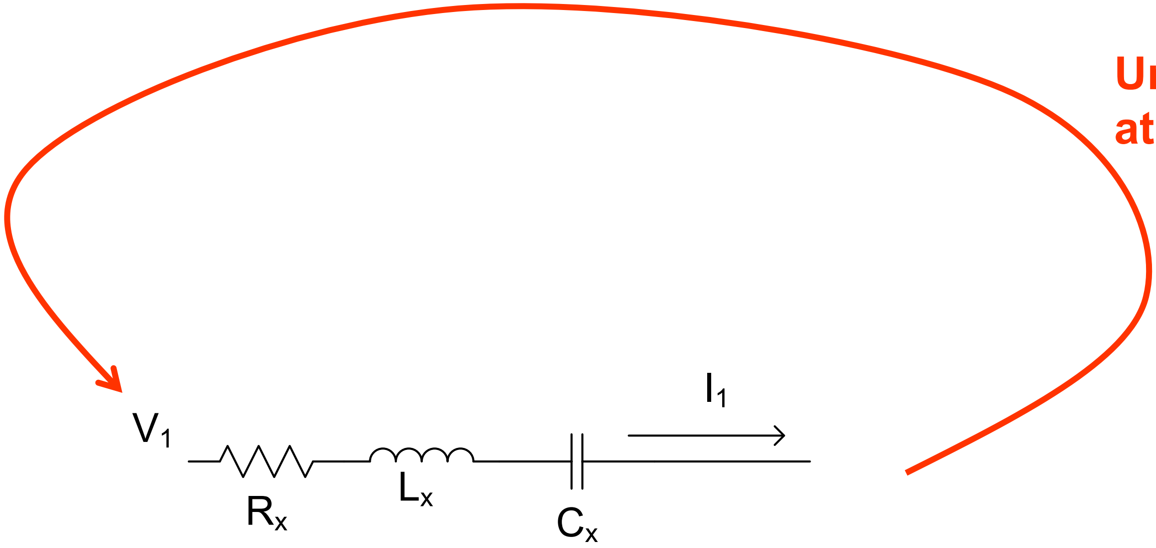
$$F_{ext} = kx + b\dot{x} + m\ddot{x}$$

Electro-Mechanical Oscillator



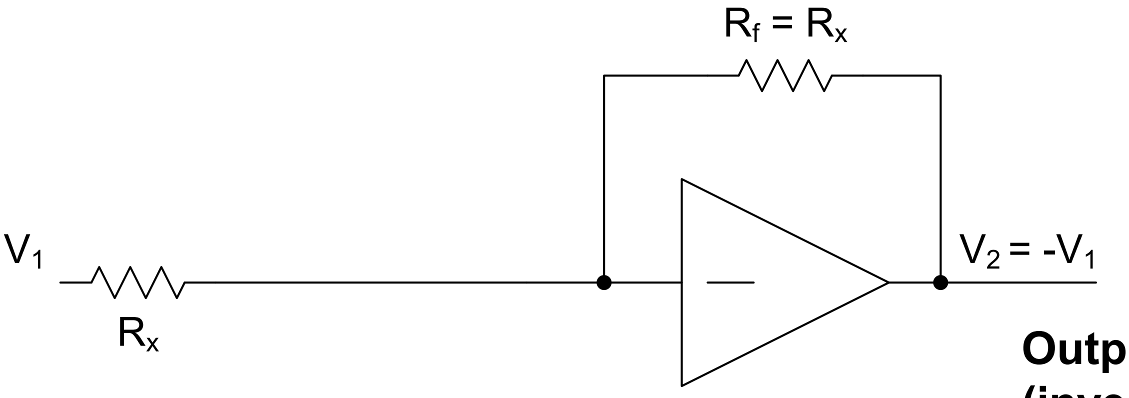
$$F_{ext} = kx + b\dot{x} + m\ddot{x}$$

Oscillator Sustaining Circuit



Unity-gain feedback at resonance

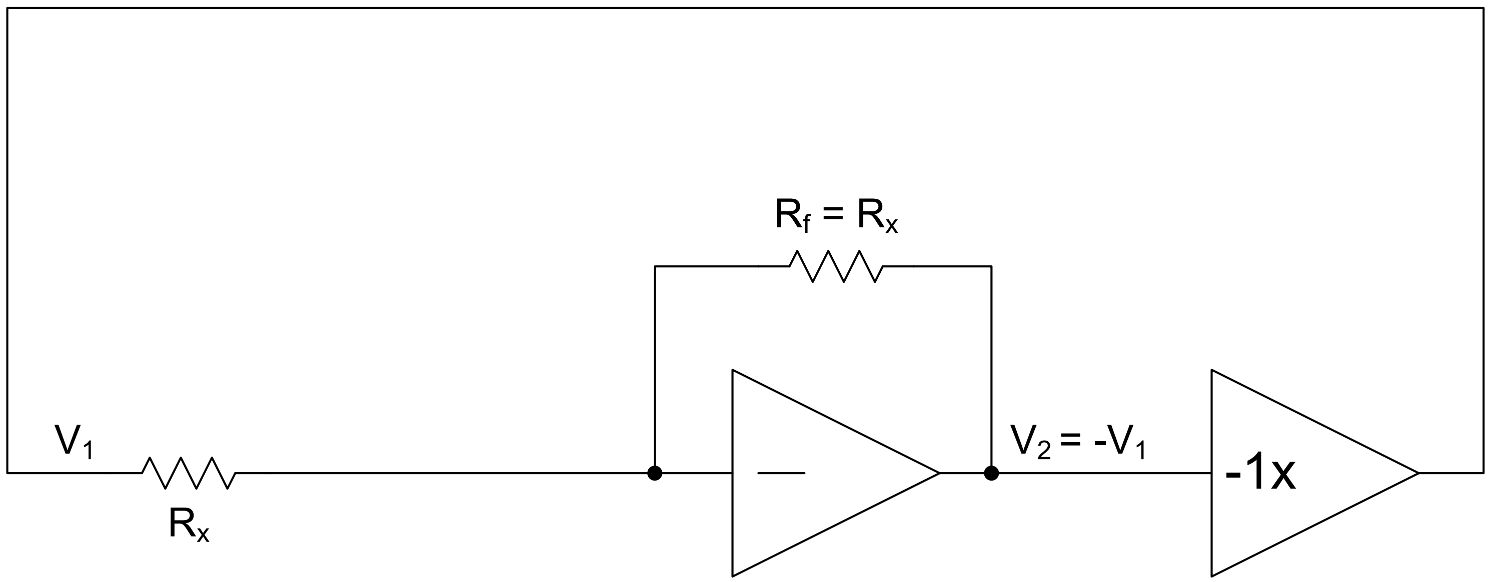
Oscillator Sustaining Circuit



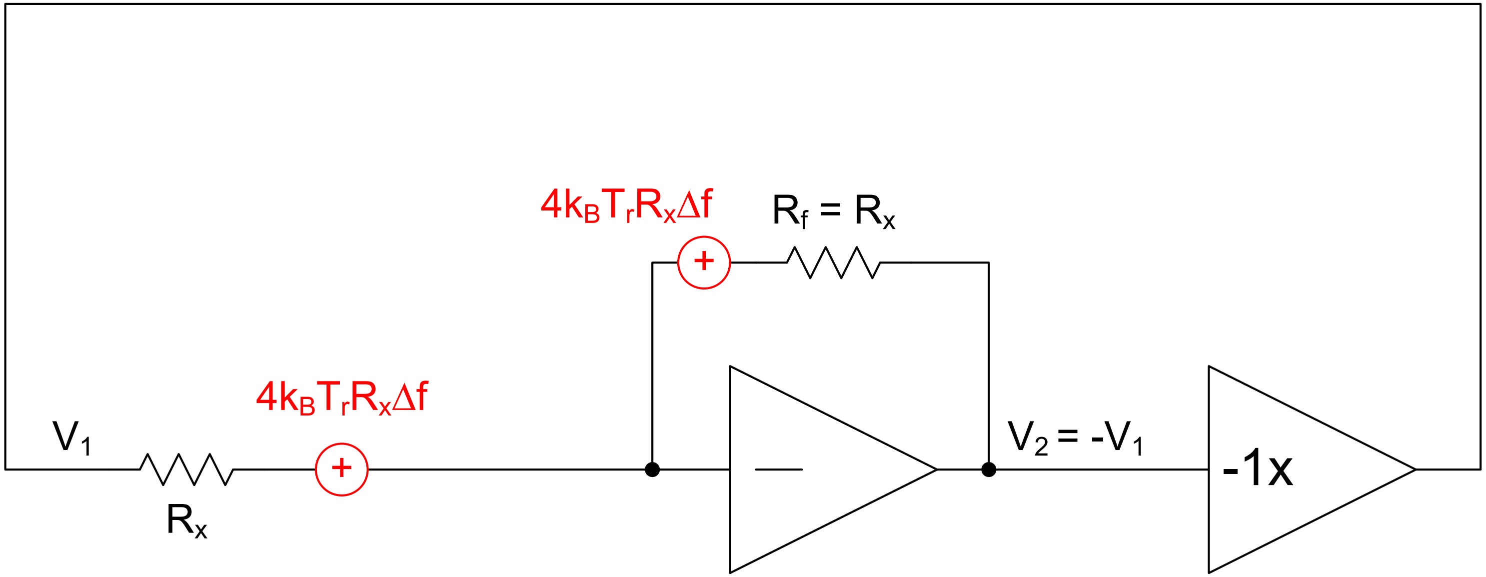
Output = Input (inverted)

Resonator at resonance

Oscillator Sustaining Circuit



Oscillator Noise Voltage



$$\overline{v_{n1}^2} = 4k_BT_RR_x\left[1+2\left(\frac{f_o}{2Q\Delta f}\right)^2\right]df$$

← **High Q?**

Phase Noise

$$L(\Delta f) = \frac{\text{noise-power}}{\text{power delivered to oscillator}}$$

$$= \frac{\overline{v_{n1}^2} / R_x}{\underbrace{k_B \overline{x^2}}_{\text{energy stored in spring}} \omega_o / Q}$$

$$= \frac{2k_B T_r}{k_B \overline{x^2} \omega_o} Q \left[1 + 2 \left(\frac{f_o}{2Q\Delta f} \right)^2 \right]$$

$$= \frac{2k_B T_r}{k_B \overline{x^2} \omega_o} \left[\underbrace{Q}_{L_{far}} + \underbrace{\frac{2}{Q} \left(\frac{f_o}{2\Delta f} \right)^2}_{L_{near}} \right]$$

$$L_{near} = \frac{4k_B T_r}{k_B \overline{x^2} \omega_o} \frac{1}{Q} \left(\frac{f_o}{2\Delta f} \right)^2$$

$$\boxed{L_{far} = \frac{2k_B T_r}{k_B \overline{x^2} \omega_o} Q}$$

→ tradeoff
(for high bandwidth optimal <u>$Q < 100$</u>)

→ for given mechanical amplitude

Phase Noise Floor – Interpretation

$$L_{far} = \frac{2k_B T_r}{k_B \overline{x^2} \omega_o} Q$$

$$= \frac{\text{Thermal Energy}}{\text{Energy Replenished by Electronics}}$$

Set by thermodynamics

$$= \frac{\text{Thermal Energy}}{\text{Energy Stored in Resonator} \big/ Q}$$

Resonator properties (size)

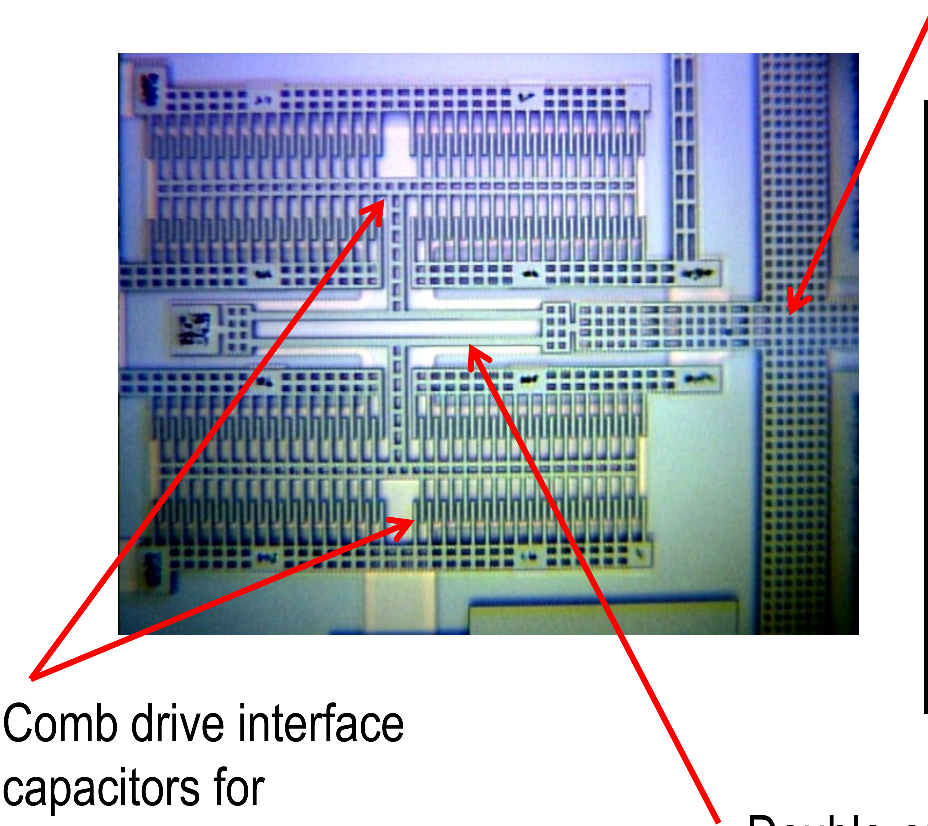
Resonator quality
Sets long-term stability

MEMS versus Quartz Resonators

1. Maximum power limited in “small” MEMS oscillators
    - Increasing resonant frequency helps
    - Bulk acoustic waves store more energy

2. High motional impedance R_x
    - Loaded Q ≈ intrinsic Q

Prototype DETF Strain Sensor



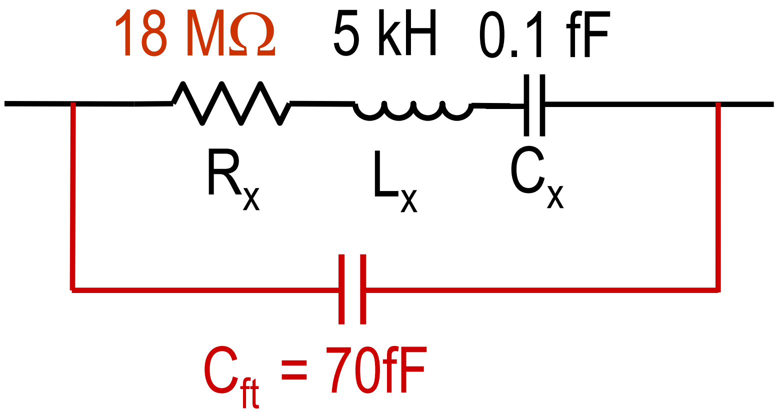
Strain Actuator

Comb drive interface capacitors for sustaining circuit

Double-ended tuning fork (DETF).

Gauge Length (L_{beam})	200 μm
Width, (W_{beam})	6 μm
Measured Resonant freq. (f_r)	217kHz
Measured Q (atmospheric pressure)	370
Measured sensitivity	38Hz/με

Problem with Low Q Operation

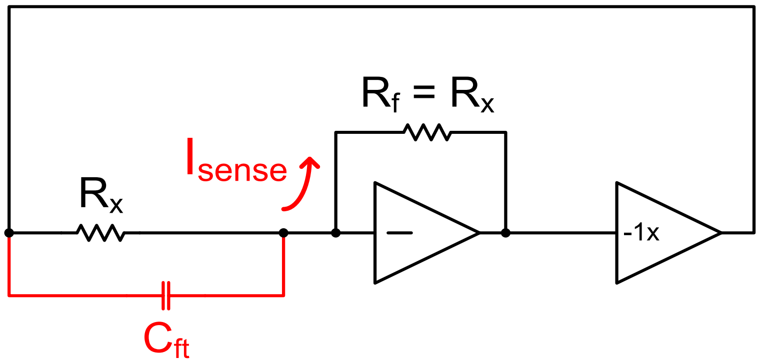


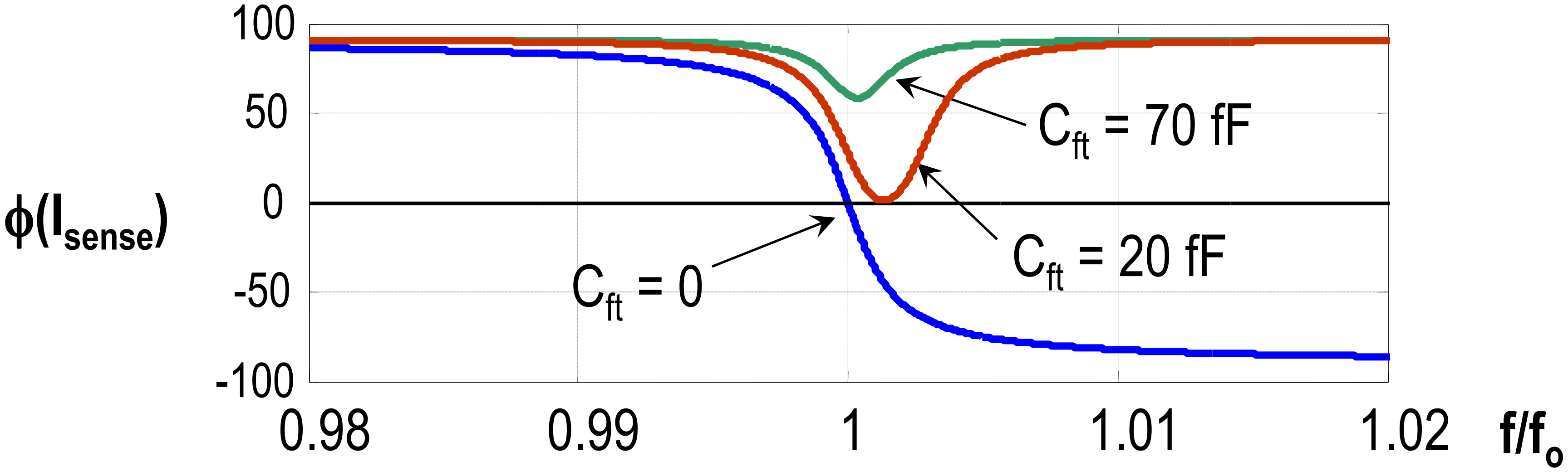
- No vacuum packaging (Q ~ 370)
- Large feature spacing ~3µm gaps
- Modest V_{bias} = 30V
- No integration: PCB parasitics

The first three items: Large R_x

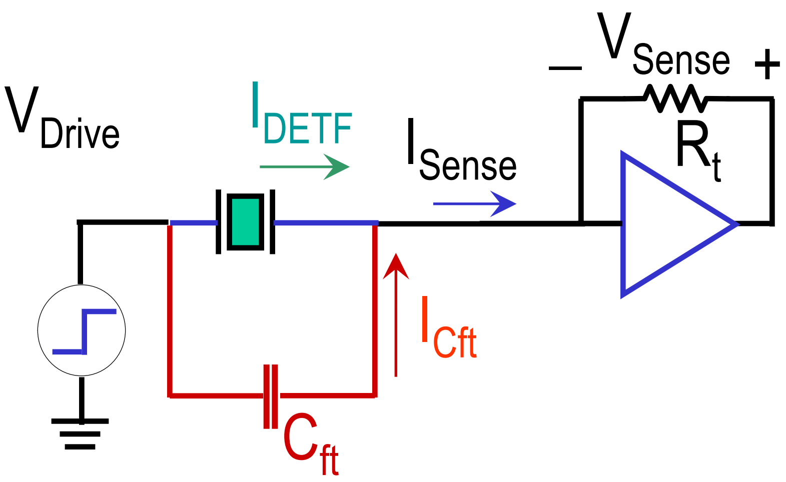
The last item: Large C_{ft}

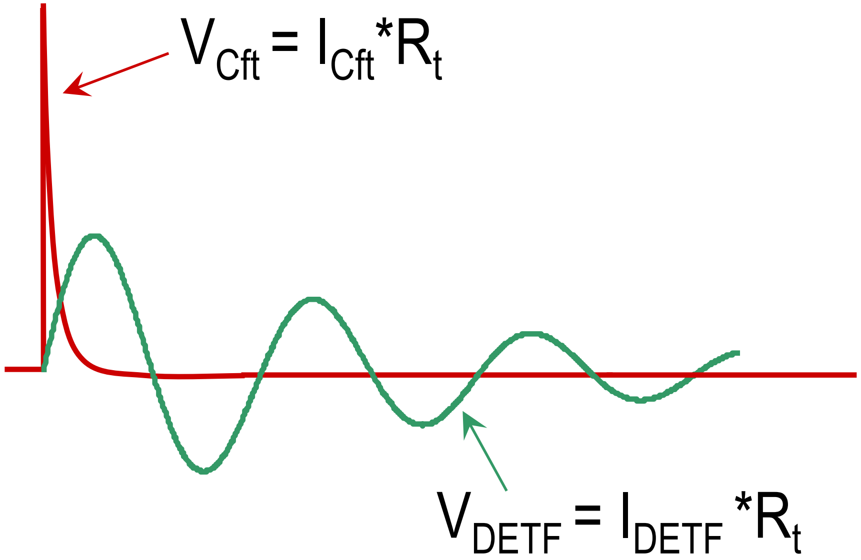
Effect of C_{ft} on Sustaining Circuit





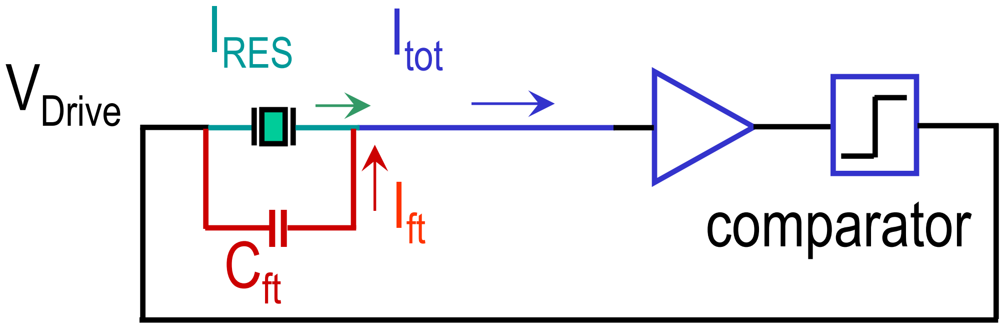
Time-Variant Sustaining Circuit

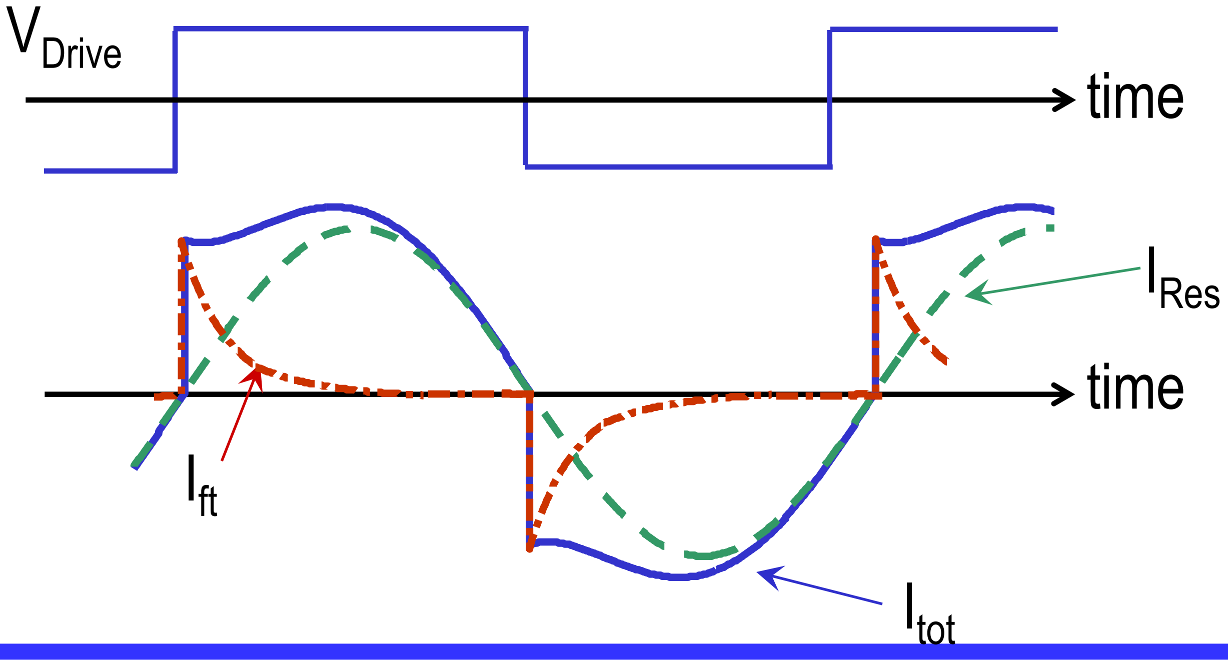




- Undesired I_{Cft} decays *quickly*
- Sinusoidal I_{DETF} decays *slowly*
- Zero crossings *set by DETF, not* C_{ft}

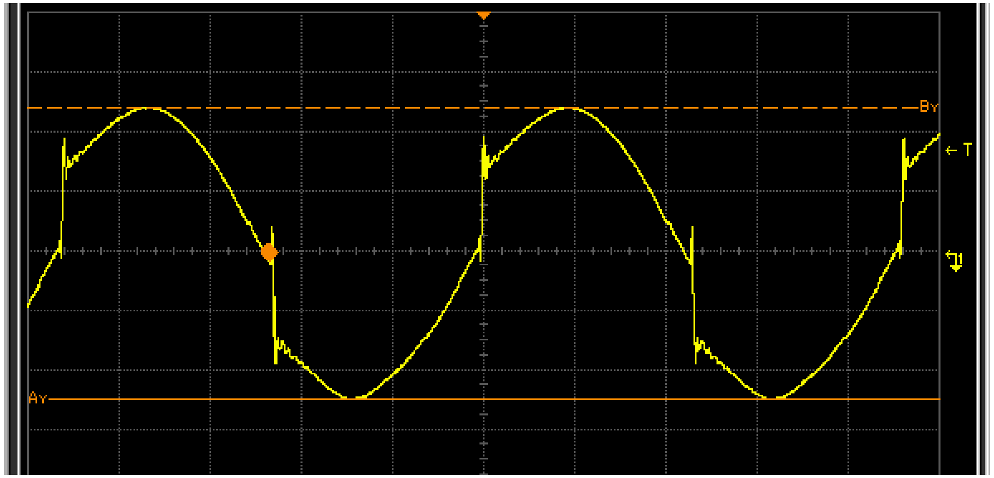
Time-Variant Oscillator (TVO)





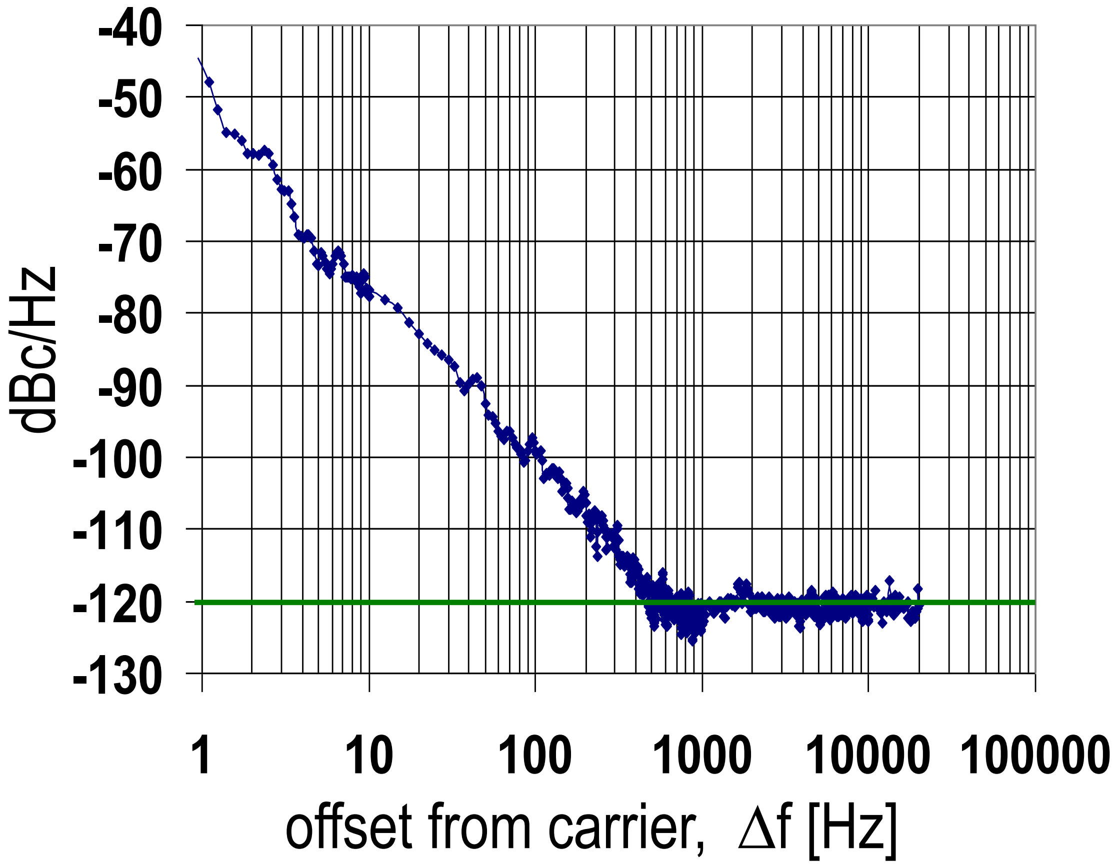
TVO Measurements

Measured Output: V_{Sense}



MEMS
resonator

Measured TVO Phase Noise

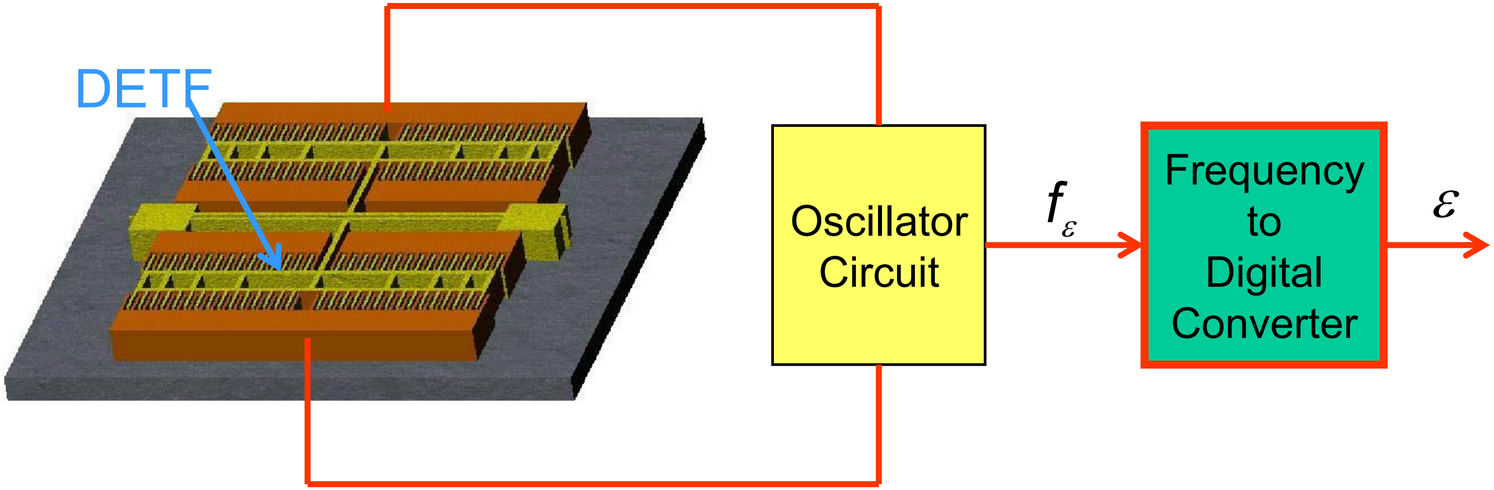


- Resolution
  - 20-nε in 10kHz
  - 200-pε/rt-Hz
  - 40-fm/rt-Hz
- Test equipment: Agilent 5501A

MEMS Oscillator Noise

	Roessig 1997	Nguyen 1999	Seshia 2002	Seth 2004	This Work 2004
Noise L_{far} [dBc/Hz]	**-70**	**-70**	**-75**	**-83**	**-120**
Vacuum	✓	✓	✓	✓	✗
Monolithic	✓	✓	✓	✗	✗
Frequency, f_o	175 kHz	16.5 kHz	145 kHz	57 kHz	217 kHz
Motional Res., R_x	?	3.8 MΩ	?	2.5 MΩ	18 M Ω
Q	17000	51000	10000	14000	370
Amplitude V_{Drive}	?	~ 34 mV	~36 mV	5.3 mV	10 V

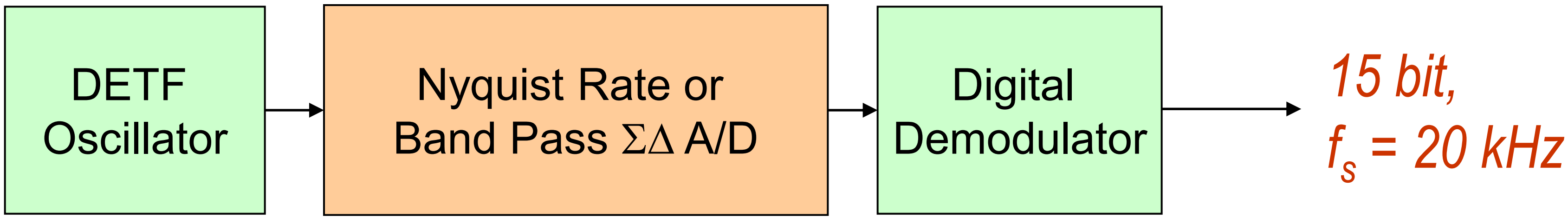
Frequency-to-Digital Conversion

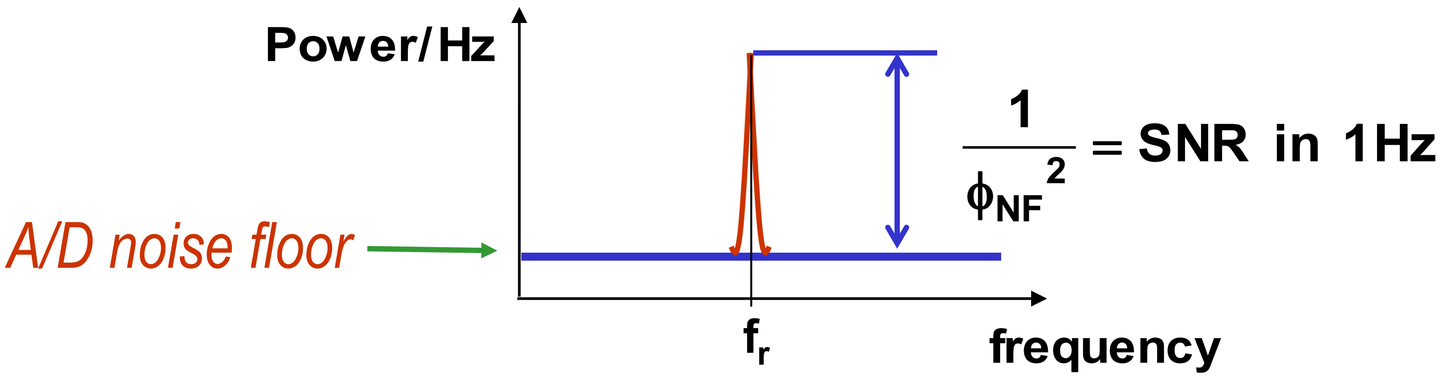


Methods:

- Digital demodulation
- Analog PLL followed by ADC
- ΣΔPLL

Digital Demodulation

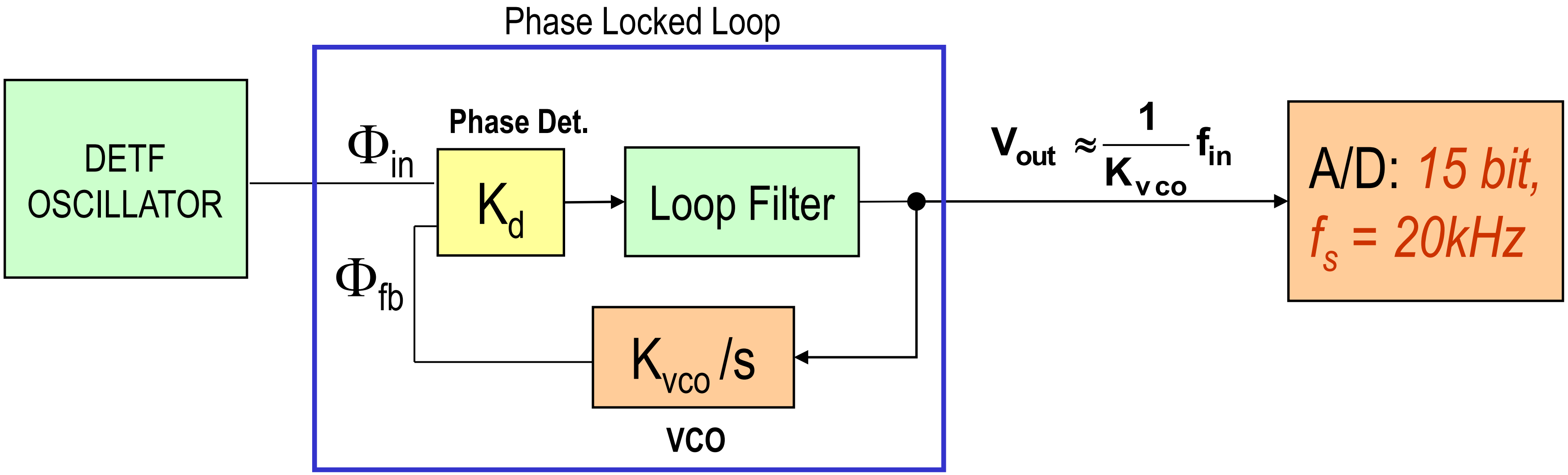




ADC Specs:

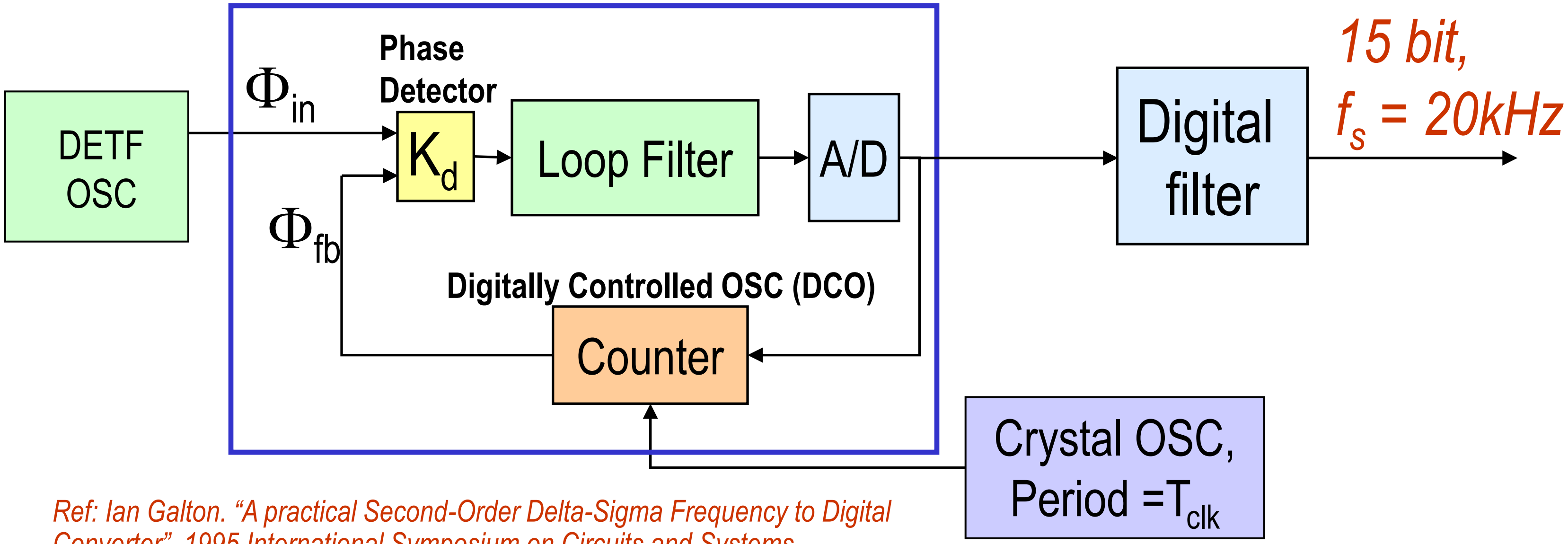
- *17 Bit A/D required*
- *Sample Rate $f_s \geq 3f_r$ = 651 kHz*
- *Bandwidth ≥ 2 x (Signal BW + Max change in f_r) = 96 kHz*

PLL Demodulator



- *Loop filter Bandwidth ≥ Signal Bandwidth (10kHz)*
- *VCO phase noise better than input phase noise (-120 dBc).*
- *Scale factor (K_{vco}) temp/supply dependence can be problem*
- *Needs extra ADC*

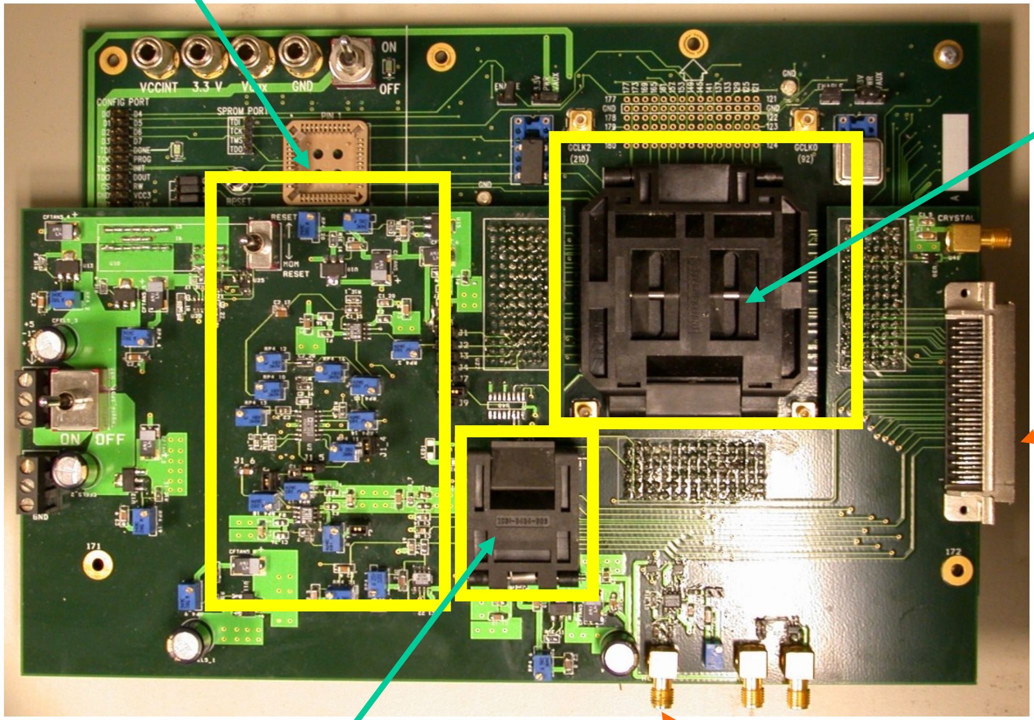
Solution: ΣΔPLL* Approach



Ref: Ian Galton. "A practical Second-Order Delta-Sigma Frequency to Digital Converter", 1995 International Symposium on Circuits and Systems.

- *Performs demodulation and A/D conversion in same step*
- *Low complexity analog filter and A/D (4 bits) for 15 bit resolution*
- *Linearity and phase noise set by crystal reference*
- *Loop filter Bandwidth = Signal Bandwidth (10kHz)*

Frequency-to-Digital Converter



Analog Filter

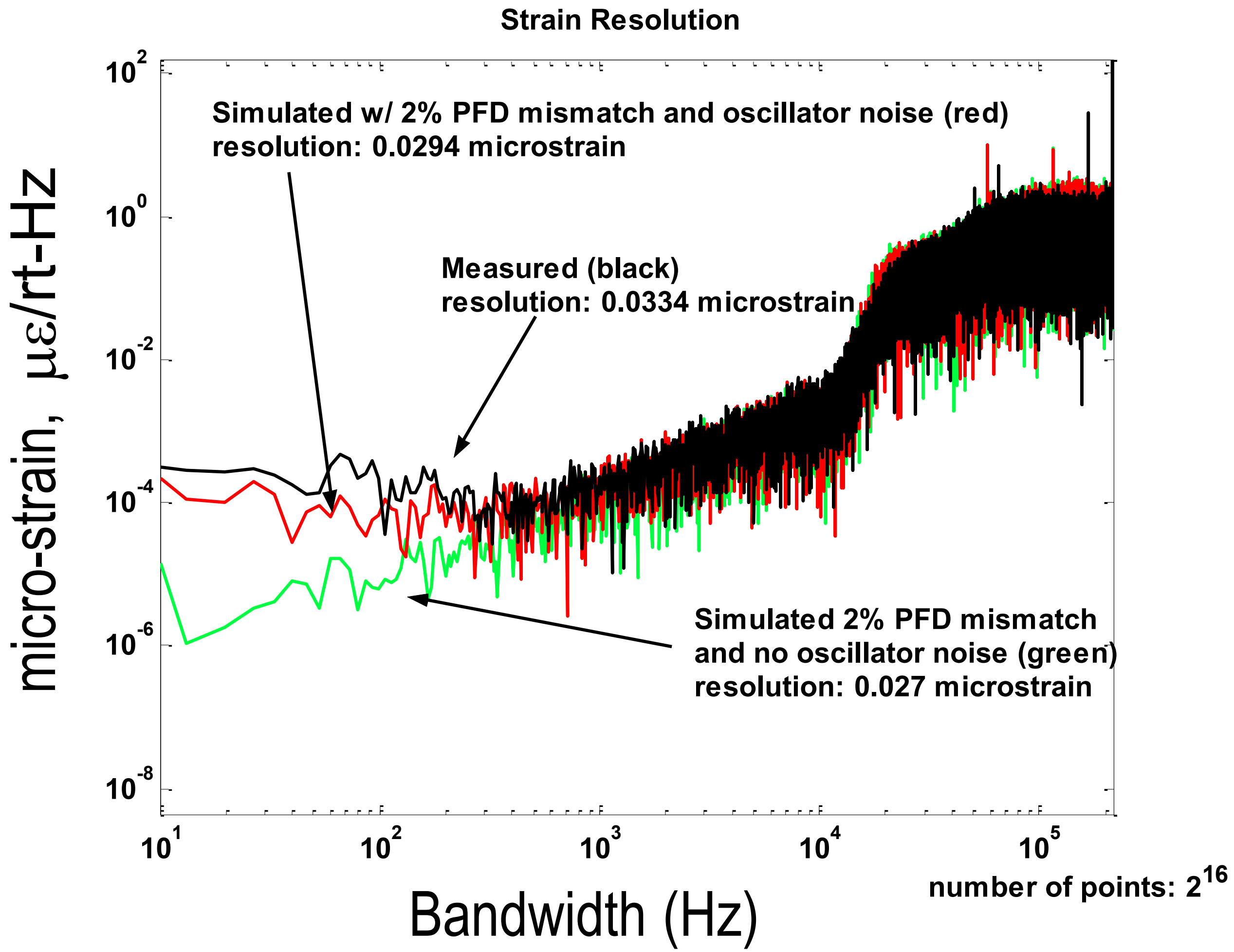
FPGA

Output:
Digitized frequency
→ strain

4bit
A/D Converter

Input from: DETF Strain
Gauge Oscillator

Measured Strain Resolution



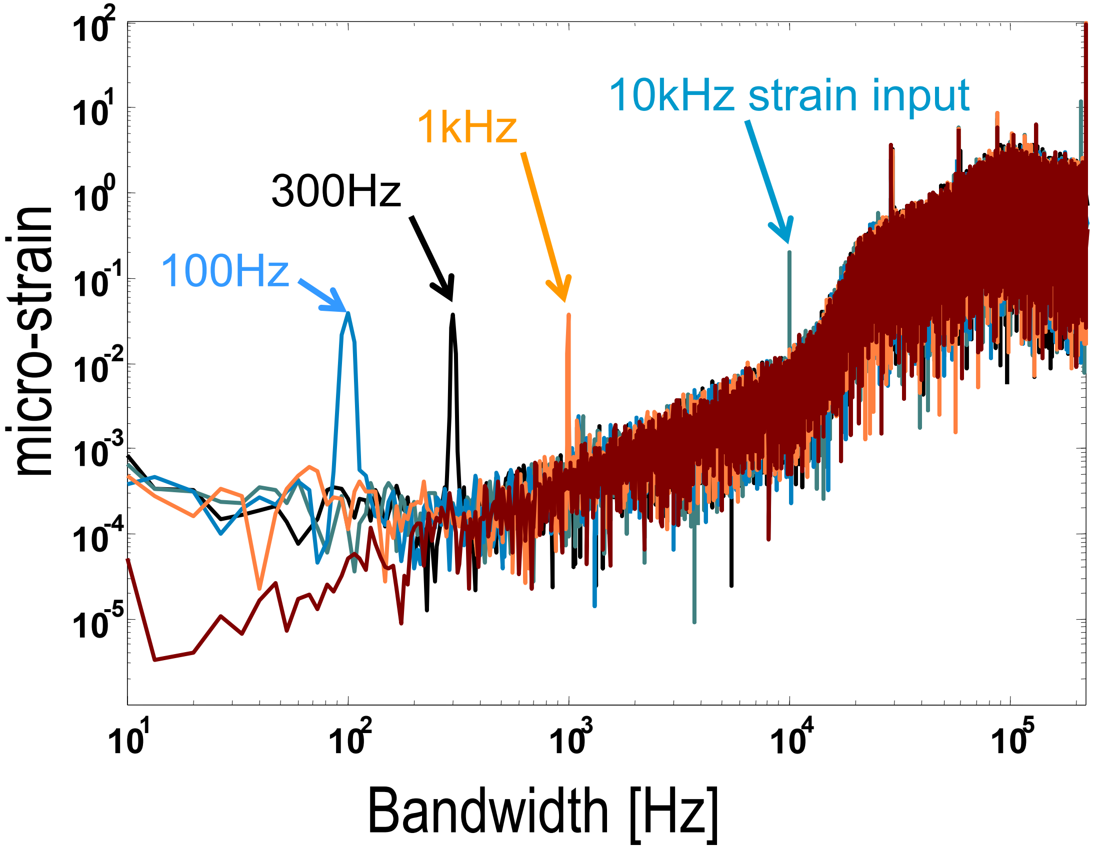
Simulated w/ 2% PFD mismatch and oscillator noise (red)
resolution: 0.0294 microstrain

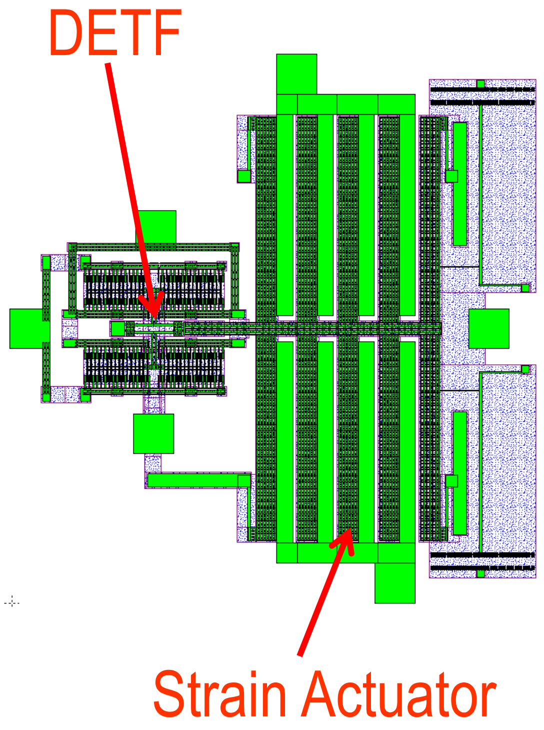
Measured (black)
resolution: 0.0334 microstrain

Simulated 2% PFD mismatch
and no oscillator noise (green)
resolution: 0.027 microstrain

number of points: 2^{16}

Test with Strain Actuator





Conclusions

- High resonator Q translates into
  - Low signal power delivered to resonator
  - High phase noise
  - Lowest phase noise is achieved for <u>moderate Q</u>
- High motional resistance, R_x
  - Typical for MEMS oscillators, exacerbated in low Q designs
  - Incompatible with conventional oscillator circuits
  - Overcome with time-variant sustaining circuit
- Strain sensor performance
  - 20-nε (rms) resolution in 10kHz
  - 200-μm gauge length
  - 40-fm/rt-Hz displacement resolution

Acknowledgements

- BSAC strain sensor research team under Professor Albert Pisano
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- Bosch for fabrication
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